Chapter 10 · Circles

WORK IT OUT

A maths workbook — concepts, worked examples, graded practice and number puzzles

Do the Learn First pages before the practice. Do the practice before the puzzles. Full solutions are at the back — but they only help you after you have tried.

Part	What it is	How to use it	Time
1	Learn First	Read and annotate. Copy every worked example by hand.	25 min
2	Worked Examples	Cover the solution. Try. Then compare line by line.	20 min
3	Guided Practice	Steps are given; you fill the gaps.	15 min
4	Practice Set A · Fluency	Straight application. Aim for speed and accuracy.	20 min
5	Practice Set B · Reasoning	Multi-step. Show every line.	25 min
6	Practice Set C · Challenge	Stretch problems. One may beat you — that is fine.	25 min
7	The Crossnumber	Answers are numbers. Wrong maths will not fit.	20 min
8	Number Search, Magic Square & Concept Bingo	Solo or with a partner.	20 min
9	Spot the Mistake	Audit a worked solution. Find the planted error.	15 min
10	Quick-Fire Round	Matching, True/False, Two Truths and a Lie.	15 min
11	The Escape Challenge	Teams of 4.	25 min

Part 1 · Learn First

A tangent touches a circle at exactly one point — and everything else in this chapter follows from that single fact, plus a right angle hiding at the point of contact.

Tangent, secant, and the point of contact

A line and a circle in the same plane can meet in only three possible ways.

Tangent. A tangent to a circle is a line that intersects the circle at only one point. That point is called the **point of contact**.

A tangent is really a limiting case of a secant: imagine a secant PQ sliding so the two points where it meets the circle move closer and closer together. When they merge into a single point, the secant has become a tangent.

Key fact: at any point on a circle, there is one and only one tangent. Through a point *inside* the circle, no tangent can be drawn — every line through it is a secant. Through a point *outside* the circle, exactly two tangents can be drawn.

Line and circle meet…	Number of common points	Name of the line
Nowhere	0	Non-intersecting line
At two points	2	Secant
At exactly one point	1	Tangent

> **Rule to remember**
>
> - A wheel rolling on the ground: the ground is a tangent to the wheel at every instant, and the spoke to that point is a radius.
> - A rope over a pulley behaves like a tangent to the circle of the pulley.

Theorem 10.1 — Tangent is perpendicular to the radius

This is the single most useful fact in the chapter: at the point of contact, the tangent and the radius always meet at 90°.

Theorem 10.1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given: a circle with centre O, and a tangent XY touching the circle at a point P. **To prove:** $OP \perp XY$.

Proof. Take any other point Q on XY and join OQ. Since XY touches the circle only at P, the point Q must lie *outside* the circle (if Q were inside, XY would cut the circle twice and become a secant). So OQ, which reaches an outside point, is longer than the radius OP: $OQ > OP$.

This is true for **every** point Q on XY other than P — so OP is the shortest distance from O to any point of the line XY. The shortest distance from a point to a line is always along the perpendicular. Hence $OP \perp XY$.

Remarks. (1) At any point on a circle there can be one and only one tangent — since the perpendicular to a given radius at that point is unique. (2) The line containing the radius through the point of contact is also called the *normal* to the circle at that point.

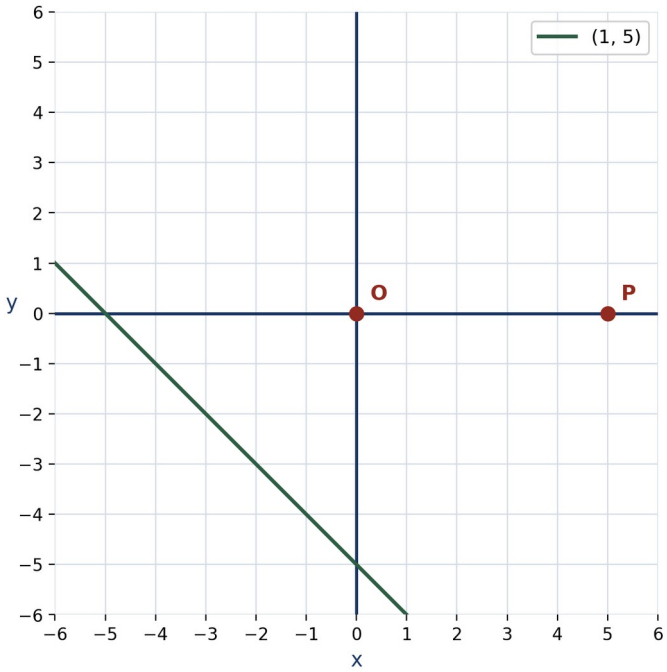

Fig. 10.5 — the radius OP is perpendicular to the tangent XY at P

Theorem 10.2 — Equal tangents from an external point

Two tangents drawn from the same outside point to a circle always turn out to be exactly the same length.

Theorem 10.2. The lengths of the two tangents drawn from an external point to a circle are equal.

Given: a circle with centre O, an external point P, and two tangents PQ and PR touching the circle at Q and R. **To prove:** $PQ = PR$.

Proof. Join OP, OQ and OR. By Theorem 10.1, the radius is perpendicular to the tangent at the point of contact, so $\angle OQP = \angle ORP = 90°$.

Now compare right triangles OQP and ORP: $OQ = OR$ (radii of the same circle), and $OP = OP$ (common side). By the RHS congruence rule, $\Delta OQP \cong \Delta ORP$, so $PQ = PR$ (CPCT).

Alternative proof, using Pythagoras. $PQ^2 = OP^2 - OQ^2 = OP^2 - OR^2 = PR^2$, so $PQ = PR$.

Bonus fact: $\angle OPQ = \angle OPR$ too — so OP bisects the angle between the two tangents. In other words, the centre always lies on the bisector of the angle formed by two tangents from an external point.

> **Two facts you will use in almost every question**
>
> - Tangent $\perp$ radius at the point of contact — this is where a right angle appears, ready for Pythagoras.
> - Tangents from the same external point are equal in length — this is where two sides of a figure suddenly become interchangeable.

Part 2 · Worked Examples

Cover the right-hand column with a sheet of paper. Try the problem yourself first. Then uncover it one line at a time and check where your reasoning parted from the model.

Example 1 — In two concentric circles, prove that a chord of the larger circle which touches the smaller circle is bisected at the point of contact

Given. Two concentric circles C_1 (larger) and C_2 (smaller) with common centre O. A chord AB of C_1 touches C_2 at P.

To prove. $AP = BP$.

Step 1. Join OP. Since AB touches the smaller circle C_2 at P, and OP is a radius of C_2 through the point of contact, Theorem 10.1 gives $OP \perp AB$.

Step 2. Now AB is also a chord of the *larger* circle C_1, and OP (through the centre) is perpendicular to it. The perpendicular from the centre of a circle to a chord always bisects the chord.

Answer: So $AP = BP$ — the chord is bisected at the point of contact.

> **Why this works**
>
> The same segment OP plays two roles at once: a radius of the small circle (giving the right angle) and a centre-to-chord perpendicular of the big circle (giving the bisection).

Example 2 — Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$

Step 1. Let $\angle PTQ = \theta$. By Theorem 10.2, $TP = TQ$, so ΔTPQ is isosceles.

Step 2. In an isosceles triangle the base angles are equal, so

$$\angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{\theta}{2}.$$

Step 3. By Theorem 10.1, $\angle OPT = 90^\circ$ (radius $\perp$ tangent at P).

Step 4. $\angle OPQ = \angle OPT - \angle TPQ = 90^\circ - \left(90^\circ - \frac{\theta}{2}\right) = \frac{\theta}{2}$.

Answer: $\frac{\theta}{2} = \frac{1}{2}\angle PTQ$, so $\angle PTQ = 2\angle OPQ$, as required.

> **Why this works**
>
> The isosceles triangle from equal tangents, combined with the right angle from Theorem 10.1, is a combination worth remembering — it appears in many angle-chasing problems.

Example 3 — PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q meet at a point T. Find the length TP

Step 1. Join OT. It meets PQ at R. Since $TP = TQ$ (Theorem 10.2), ΔTPQ is isosceles and TO bisects $\angle PTQ$, so $OT \perp PQ$ and R is the midpoint of PQ.

Step 2. $PR = RQ = 4$ cm (half of the 8 cm chord).

Step 3. In right triangle OPR: $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$ cm.

Step 4. $\angle TPR + \angle RPO = 90° = \angle TPR + \angle PTR$, so $\angle RPO = \angle PTR$. Hence right triangles TRP and PRO are similar (AA).

Step 5. $\frac{TP}{PO} = \frac{RP}{RO}$, i.e. $\frac{TP}{5} = \frac{4}{3}$.

Answer: $TP = \frac{20}{3}$ cm.

> **Why this works**
>
> This question can also be solved with two applications of Pythagoras (letting $TP = x$, $TR = y$, and using both right triangles TRP and OPT) — both methods appear in the textbook and are worth practising.

Example 4 — The length of a tangent from a point A at distance 5 cm from the centre of a circle is 4 cm. Find the radius of the circle

Step 1. Let the tangent touch the circle at P. By Theorem 10.1, $OP \perp AP$, so ΔOPA is right-angled at P.

Step 2. By Pythagoras: $OA^2 = OP^2 + AP^2$, i.e. $5^2 = OP^2 + 4^2$.

Step 3. $25 = OP^2 + 16$, so $OP^2 = 9$.

Answer: $OP = 3$ cm — the radius of the circle.

> **Why this works**
>
> This is the standard triangle for the whole chapter: (distance to centre)² = (radius)² + (tangent length)². Learn to spot it instantly.

Part 3 · Guided Practice

The method is laid out for you. Fill in the blanks. If you can do these without looking back at Part 2, you are ready for Part 4.

1. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find the length PQ.

Since PQ is a tangent and OP is the radius to the point of contact, $\angle OPQ = ¿$ _____ °.

So ΔOPQ is right-angled at P, and by Pythagoras: $OQ^2 = OP^2 + PQ^2$.

$12^2 = 5^2 + PQ^2$, so $PQ^2 = 144 - ¿¿$.

$PQ = ¿$ _____ cm.

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2. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius.

Let the tangent touch the circle at P. $\angle OPQ = ¿$ _____ ° (tangent $\perp$ radius).

By Pythagoras: $OQ^2 = OP^2 + QP^2$, so $25^2 = OP^2 + ¿^2$.

$625 = OP^2 + 576$, so $OP^2 = ¿$.

Radius $¿$ _____ cm.

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3. Two concentric circles have radii 5 cm and 3 cm. Find the length of the chord of the larger circle that touches the smaller circle.

Let the chord AB of the larger circle touch the smaller circle at P. Join OP; then $OP \perp AB$ and $OP = ¿$ _____ cm (radius of smaller circle).

In right triangle OPA, $OA = ¿$ _____ cm (radius of larger circle), so $AP^2 = OA^2 - OP^2 = ¿ - ¿¿$.

$AP = ¿$ _____ cm, and since OP bisects the chord, $AB = 2 \times AP = ¿$ _____ cm.

Practice Set A · Fluency

Straight application of the method. Aim to get every one right, then aim to get them right quickly.

#	Question	Your working and answer
1	How many tangents can a circle have?	
2	Fill in the blanks: (i) A tangent to a circle intersects it in _____ point(s). (ii) A line intersecting a circle in two points is called a _____. (iii) A circle can have _____ parallel tangents at the most. (iv) The common point of a tangent to a circle and the circle is called the _____.	
3	A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Choose the correct length of PQ: (A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm.	
4	Draw a circle and two lines parallel to a given line such that one is a tangent and the other a secant to the circle.	
5	The length of a tangent from a point A at distance 5 cm from the centre of a circle is 4 cm. Find the radius of the circle.	
6	In Fig., if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, find $\angle PTQ$.	
7	If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, find $\angle POA$.	

Practice Set B · Reasoning

Two or more steps, or a step you have to choose for yourself. Show every line — marks live in the working.

#	Question	Your working and answer
1	Prove that the tangents drawn at the ends of a diameter of a circle are parallel.	
2	Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.	
3	Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.	
4	A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB+CD=AD+BC$.	
5	XY and $Xprime Yprime$ are two parallel tangents to a circle with centre O, and another tangent AB with point of contact C intersects XY at A and $Xprime Yprime$ at B. Prove that $\angle AOB=90^\circ$.	
6	Prove that the parallelogram circumscribing a circle is a rhombus.	

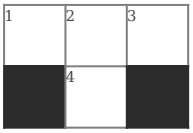
Practice Set C · Challenge

Harder, and deliberately so. If one defeats you, write down exactly where you got stuck. That sentence is worth more than a copied answer.

#	Question	Your working and answer
1	Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.	
2	A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.	
3	Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.	
4	Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.	
5	PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.	

Part 7 · The Crossnumber

Every answer is a number. One digit per square. Because answers cross, a wrong calculation simply will not fit — this puzzle marks itself.



ACROSS

1. $12^2 - 5^2$ (Ex. 10.1 Q3: OQ=12, radius=5, PQ²) *(3 digits)*

4. Radius when tangent from A (5 cm away) is 4 cm *(1 digits)*

DOWN

2. Hypotenuse of the classic 5–12–13 right triangle *(2 digits)*

3. $OA^2 - OP^2$ when OA=5, OP=4 (i.e. $5^2 - 4^2$) *(1 digits)*

Bonus questions that did not fit the grid — work them out anyway: Radius when tangent = 24 cm, distance to centre = 25 cm; Chord of the larger circle touching the smaller circle (radii 5 and 3); From tangent length problem, $\frac{20}{3} \times 3$ — numerator of TP; OQ squared minus tangent squared, when OQ=25, tangent=24: $25^2 - 24^2$ ... take OQ itself; Length of a tangent from A when OA=5 and radius=3; AP^2 when OA=5, OP=3 (i.e. $5^2 - 3^2$)

Part 8 · Number Search & Magic Square

Work out each answer below, then find it hidden in the digit grid — across, down, diagonally or backwards.

2	6	8	8	2	4	1	5	2	6	9	6	9
3	0	3	0	3	8	7	7	2	2	1	2	1
2	9	3	4	3	8	9	2	6	1	8	2	3
2	1	3	8	3	9	7	5	9	2	3	8	6
8	3	1	2	0	9	0	5	4	1	6	0	2
8	2	1	9	6	8	0	2	2	4	7	0	2
8	8	2	2	2	1	3	0	5	3	9	5	4
3	2	1	9	1	1	4	0	1	3	3	8	5
5	5	4	5	4	0	2	6	4	1	7	3	0
5	2	4	4	5	6	8	7	0	9	9	7	4
1	8	0	1	1	7	7	3	1	4	9	0	4
4	8	2	0	6	5	8	3	3	1	5	7	1
3	5	2	0	1	8	2	7	8	3	4	8	4

☐ 119	☐ 20	☐ 13	☐ 25	☐ 16
☐ 7	☐ 3	☐ 8	☐ 9	☐ 4

Magic Square — every row, column and diagonal adds to the same total.

8	**1**	
3		
	9	**2**

Magic constant: 15. Fill the empty cells.

Part 9 · Spot the Mistake

Each solution below reaches a wrong answer, and exactly one step is to blame. Find the first wrong step and rewrite it correctly. Marking other people's work is the fastest way to stop making their mistakes.

🔍 Spot the Mistake

Finding PQ when radius = 5 cm, OQ = 12 cm

Step 1. Since PQ is tangent, $\angle OPQ = 90^\circ$.

Step 2. By Pythagoras, $OQ^2 = OP^2 + PQ^2$, so $PQ^2 = OQ^2 + OP^2 = 144 + 25 = 169$.

Step 3. $PQ = 13$ cm.

_____ The first wrong step is Step _____ . What should it say?

_____

🔍 Spot the Mistake

Tangent length from a point A, OA = 5 cm, radius = 4 cm

Step 1. ΔOPA is right-angled at P, since tangent $\perp$ radius.

Step 2. $AP^2 = OA^2 + OP^2 = 25 + 16 = 41$.

Step 3. $AP = \sqrt{41}$ cm.

_____ The first wrong step is Step _____ . What should it say?

_____

🔍 Spot the Mistake

Number of tangents from a point inside a circle

Step 1. A point P is taken inside the circle.

Step 2. Every line drawn through P meets the circle in two points.

Step 3. So exactly one tangent can be drawn from P.

_____ The first wrong step is Step _____ . What should it say?

_____

Part 10 · Quick-Fire Round

Match each quantity to its value (all from the standard 5 cm radius / 12 cm, 13 cm, 4 cm, 3 cm tangent problems)

Draw a line from each item on the left to its match on the right.

#	Left	Right (shuffled)
1	Tangent length when OQ=12, radius=5	$\sqrt{119}$ cm
2	Radius when tangent=24, OQ=25	3 cm
3	Radius when tangent=4, OA=5	2
4	Chord of larger circle (radii 5, 3)	7 cm
5	∠ between tangent and radius at contact	90°
6	Number of tangents from an external point	8 cm

True or False Sprint

Read each statement to yourself (or take turns with a partner). Decide True or False before you check the solutions.

#	Statement	Your call
1	A tangent to a circle intersects it in exactly one point.	☐ True ☐ False
2	A secant intersects a circle in exactly one point.	☐ True ☐ False
3	From a point inside a circle, exactly one tangent can be drawn.	☐ True ☐ False
4	The tangent at a point of a circle is perpendicular to the radius through that point.	☐ True ☐ False
5	The lengths of two tangents from an external point to a circle are always equal.	☐ True ☐ False
6	A circle can have infinitely many parallel tangents.	☐ True ☐ False
7	The centre of a circle lies on the bisector of the angle between two tangents drawn from an external point.	☐ True ☐ False
8	A tangent can be thought of as a secant whose two points of intersection have merged into one.	☐ True ☐ False

Two Truths and a Lie

Two statements are true, one is false. Circle the lie — then correct it.

Set 1

(a) A tangent meets a circle at exactly one point.

(b) A secant meets a circle at exactly two points.

(c) From a point inside a circle, two tangents can be drawn.

The lie is _____ . Corrected: _____

Set 2

(a) The tangent is perpendicular to the radius at the point of contact.

(b) Tangents from an external point are always equal in length.

(c) Tangents from an external point are perpendicular to each other.

The lie is _____ . Corrected: _____

Set 3

(a) A point on the circle has exactly one tangent through it.

(b) OP bisects the angle between the two tangents from an external point P.

(c) A circle can have three parallel tangents.

The lie is _____ . Corrected: _____

Part 11 · The Escape Challenge

Five gates, each a calculation. Each gives you one digit. Collect all five to unlock the exit code.

🔒 GATE 1 — The Contact Gate

How many common points does a tangent have with a circle?

Digit: ☐

🔒 GATE 2 — The Angle Gate

What is $\angle OPQ$ in degrees, where OP is a radius and PQ is the tangent at P?

Digit: ☐

🔒 GATE 3 — The Pythagoras Gate

Radius = 5 cm, distance to external point = 13 cm. Find the tangent length.

Digit: ☐

🔒 GATE 4 — The Equal-Tangents Gate

From an external point, how many tangents can be drawn to a circle?

Digit: ☐

🔒 GATE 5 — The Radius Gate

Tangent length = 24 cm, distance to centre = 25 cm. Find the radius.

Digit: ☐

EXIT CODE: ☐ ☐ ☐ ☐ ☐

Formula Sheet

Cut this page out and keep it. Everything in this chapter, on one page.

Tangent-radius right angle

$$\angle OPT = 90^\circ \quad (O = \text{centre}, P = \text{point of contact})$$

Tangent length (Pythagoras)

$$OT^2 = OP^2 + PT^2$$

Equal tangents

$$PT_1 = PT_2 \quad (\text{tangents from the same external point } T)$$

Angle sum in tangent triangle

$$\angle PTQ + \angle POQ = 180^\circ$$

Isosceles tangent triangle

$$\angle TPQ = \angle TQP = 90^\circ - \frac{1}{2}\angle PTQ$$

Full Solutions

Not just answers — working. Compare your method with this one; if they differ but both reach the right answer, make sure you understand why yours also works.

Part 3 · Guided Practice

1. 90°; $PQ^2 = 144 - 25 = 119$; PQ = √119 cm · **2.** 90°; 24; $OP^2 = 49$; radius = 7 cm · **3.** OP = 3; OA = 5; $AP^2 = 25 - 9 = 16$; AP = 4 cm; AB = 8 cm

Practice Set A

1. Infinitely many tangents can be drawn to a circle, one at every point on it.

2. (i) one (ii) secant (iii) two (iv) point of contact.

3. (D) — $PQ = \sqrt{OQ^2 - OP^2} = \sqrt{144 - 25} = \sqrt{119}$ cm.

4. Draw the given line l; draw a circle touching l (one tangent) and a parallel line through the circle cutting it twice (a secant) on either side.

5. ΔOPA right-angled at P: $OP^2 = OA^2 - AP^2 = 25 - 16 = 9$, so radius ¿ 3 cm.

6. $\angle PTQ = 180^\circ - \angle POQ = 180^\circ - 110^\circ = 70^\circ$ (angles OPT, OQT are 90° each, angle sum of quadrilateral OPTQ is 360°).

7. $\angle POA = \frac{1}{2}(180^\circ - 80^\circ) = 50^\circ$, since $\Delta OAP \cong \Delta OBP$ and OP bisects $\angle APB$; using the quadrilateral OAPB, $\angle AOB = 180^\circ - 80^\circ = 100^\circ$, so $\angle POA = 50^\circ$.

Practice Set B

1. Tangents at ends of a diameter are both perpendicular to the same diameter (Theorem 10.1), so they are perpendicular to the same line — hence parallel to each other.

2. The perpendicular at the point of contact is, by Theorem 10.1 (converse), along the radius, and every radius passes through the centre.

3. Chord ¿ $2\sqrt{5^2 - 3^2} = 2\sqrt{16} = 8$ cm.

4. Using equal tangents from each vertex: $AB + CD = (AP + PB) + (CR + RD)$ and $AD + BC = (AS + SD) + (BQ + QC)$; since $AP = AS$, $BP = BQ$, $CQ = CR$, $DR = DS$ (tangents from each vertex are equal), both sums equal $AP + PB + CR + RD$, so $AB + CD = AD + BC$.

5. $OC \perp AB$ (radius ⊥ tangent) and $\angle OCA = \angle OCB = 90^\circ$; since $XY \parallel Xprime\,Yprime$, using the equal-tangent triangles at A and B and the angle sum, $\angle AOC + \angle BOC = 90^\circ$, i.e. $\angle AOB = 90^\circ$.

6. In a parallelogram circumscribing a circle, tangents from each vertex are equal, giving $AB + CD = AD + BC$; combined with $AB = CD$ and $AD = BC$ (parallelogram), this forces $AB = BC = CD = AD$, so it is a rhombus.

Practice Set C

1. $\angle OPT + \angle OQT = 180^\circ$ (each 90°) in quadrilateral OPTQ, so $\angle PTQ + \angle POQ = 360^\circ - 180^\circ = 180^\circ$ — supplementary.

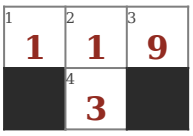
2. Let the incircle touch AB at F and AC at E, with $AF = AE = x$ (equal tangents from A). Then $BF = BD = 8$ and $CE = CD = 6$ (equal tangents from B and C), so $AB = x + 8$, $AC = x + 6$, $BC = 14$. Semi-perimeter $s = x + 14$. Area ¿ $r \cdot s = 4(x + 14)$; by Heron's formula, Area ¿ $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(x+14)(6)(8)(x)}$. Squaring: 16 ¿, so $16(x + 14) = 48x$, giving $x = 7$. Hence $AB = 15$ cm and $AC = 13$ cm.

3. Using equal tangent pairs at each vertex of quadrilateral ABCD, $\angle AOB + \angle COD = 180^\circ$ and $\angle BOC + \angle AOD = 180^\circ$, by the same congruent-triangle argument as Theorem 10.2 applied at each vertex.

4. See Worked Example 2: using the isosceles triangle TPQ and $\angle OPT = 90^\circ$, $\angle OPQ = \frac{1}{2}\angle PTQ$, i.e. $\angle PTQ = 2\angle OPQ$.

5. See Worked Example 3: $OR = 3$ cm, then similar triangles TRP and PRO give $TP = \frac{20}{3}$ cm.

Part 7 · Crossnumber — Solved Grid



Across: 1 = 119 · 4 = 3
Down: 2 = 13 · 3 = 9

Part 8 · Number Search — Solved Grid

2	6	8	8	2	4	1	5	2	6	9	6	9
3	0	3	0	3	8	7	7	2	2	1	2	1
2	9	3	4	3	8	9	**2**	6	1	8	2	3
2	**1**	**3**	8	3	9	7	**5**	9	2	3	8	6
8	3	1	2	0	9	0	5	4	1	6	0	2
8	2	1	9	6	8	0	**2**	2	4	7	0	2
8	**8**	2	2	2	1	3	**0**	5	3	**9**	5	4
3	2	1	**9**	**1**	**1**	4	0	1	3	3	8	5
5	5	**4**	5	4	0	2	6	4	1	7	3	0
5	2	4	4	5	6	8	7	0	9	9	7	4

1	8	0	1	**1**	7	7	3	1	4	9	0	4
4	8	2	0	**6**	5	8	3	3	1	5	7	1
3	5	2	0	1	8	2	**7**	8	3	4	8	4

Part 8 · Magic Square — Solved (constant 15)

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

Part 9 · Spot the Mistake

Problem	First wrong step	It should say
Finding PQ when radius = 5 cm, OQ = 12 cm	Step 2	Pythagoras needs a **subtraction**, not addition, because OQ is the hypotenuse: $PQ^2 = OQ^2 - OP^2 = 144 - 25 = 119$, so $PQ = \sqrt{119}$ cm, not 13.
Tangent length from a point A, OA = 5 cm, radius = 4 cm	Step 2	OA is the hypotenuse (it joins the external point to the centre), so it must be **subtracted from**, not added to: $AP^2 = OA^2 - OP^2 = 25 - 16 = 9$, so $AP = 3$ cm.
Number of tangents from a point inside a circle	Step 3	If every line through P is a secant (meets the circle twice), then **no** tangent can be drawn from a point inside the circle — the count is 0, not 1.

Part 10 · Matching

#	Left	Correct match
1	Tangent length when OQ=12, radius=5	$\sqrt{119}$ cm
2	Radius when tangent=24, OQ=25	7 cm
3	Radius when tangent=4, OA=5	3 cm
4	Chord of larger circle (radii 5, 3)	8 cm

#	Left	Correct match
5	∠ between tangent and radius at contact	90°
6	Number of tangents from an external point	2

Part 10 · True or False

1-T · 2-F · 3-F · 4-T · 5-T · 6-F · 7-T · 8-T

Part 10 · Two Truths and a Lie

Set	The lie	Correction
1	c	No tangent at all can be drawn from a point inside a circle — every line through it is a secant.
2	c	Nothing guarantees the two tangents are perpendicular — that angle depends on the position of the external point; only their equal length is guaranteed.
3	c	A circle can have at most **two** parallel tangents (one on each side) — never three.

Part 11 · The Escape Challenge

EXIT CODE: 1 · 9 · 1 · 2 · 7

(Answer: 1) (Answer: 90) (Answer: 12) (Answer: 2) (Answer: 7)

One right angle — tangent meets radius at 90° — and one pair of equal segments — tangents from an external point — unlock every question in this chapter. Spot them, and the rest is Pythagoras and congruent triangles. ⊙